Below are examples of problems I might ask on a final exam, organized by function. At the end of the guide are practice problems to find the solutions to systems of equations.

**PLEASE NOTE:** The questions on your exam will ask you to relate the functions to each other, so make sure that you understand the relationships between them.

**EXPONENTIAL FUNCTIONS:**

1. Rationalize the denominator.
2. Simplify
3. Consider the equation . Select **all** that apply.
   1. -3
   2. -1
   3. -½
   4. 1
   5. 3
4. Simplify the expression. Remember, no negative exponents should be remaining.
   1. 
   2. 
   3. 
   4. 
   5. 
   6. 
5. Without graphing, identify if the following functions represent exponential **growth** or **decay**. **EXPLAIN** how you know.
   1. 
   2. 
   3. 
6. Complete the table for . Then sketch the function below.

|  |  |
| --- | --- |
| *x* | *f(x)* |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

What is the domain and range of the function?

1. Complete the table for . Then sketch the function below.

|  |  |
| --- | --- |
| *x* | *f(x)* |
| -2 |  |
| -1 |  |
| 0 |  |
| 1 |  |
| 2 |  |

1. What is the domain and range of the function ?
2. Compare the graph of the parent function, , to the graph of . Describe how the graph has shifted.
3. Compare the graph of the parent function, , to the graph of . Describe how the graph has shifted.
4. Rewrite the equation  in a simpler form so that it is represented by only two transformations of .
5. Rewrite the equation  in a simpler form so that it is represented by only two transformations of .
6. A scientist places 7.35 grams of a radioactive element in a dish. The half-life of the element is 2 days. After *d* days, the number of grams of the element remaining in the dish is given by the function *R(d)* = 7.35(½)*d*/2. Which statement is true about the equation when it is rewritten without a fractional exponent? Choose **all** that apply.
7. An approximately equivalent equation is *R(d)* = 7.35(0.250)*d*.
8. An approximately equivalent equation is *R(d)* = 7.35(0.707)*d*.
9. The base of the exponent in this form of the equation can be interpreted to mean that the element decays by 0.250 grams per day.
10. The base of the exponent in this form of the equation can be interpreted to mean that the element decays by 0.707 grams per day.
11. The base of the exponent in this form of the equation can be interpreted to mean that about 25% of the element remains from one day to the next day.
12. The base of the exponent in this form of the equation can be interpreted to mean that about 70.7% of the element remains from one day to the next day.

**LOGARITHMIC FUNCTIONS:**

1. Write in log form:





1. Write in exponential form:

1. Simplify:

1. Evaluate using a calculator. Round to 3 decimal places.

1. Expand by using the properties of logs to rewrite each expression as the sum or difference:

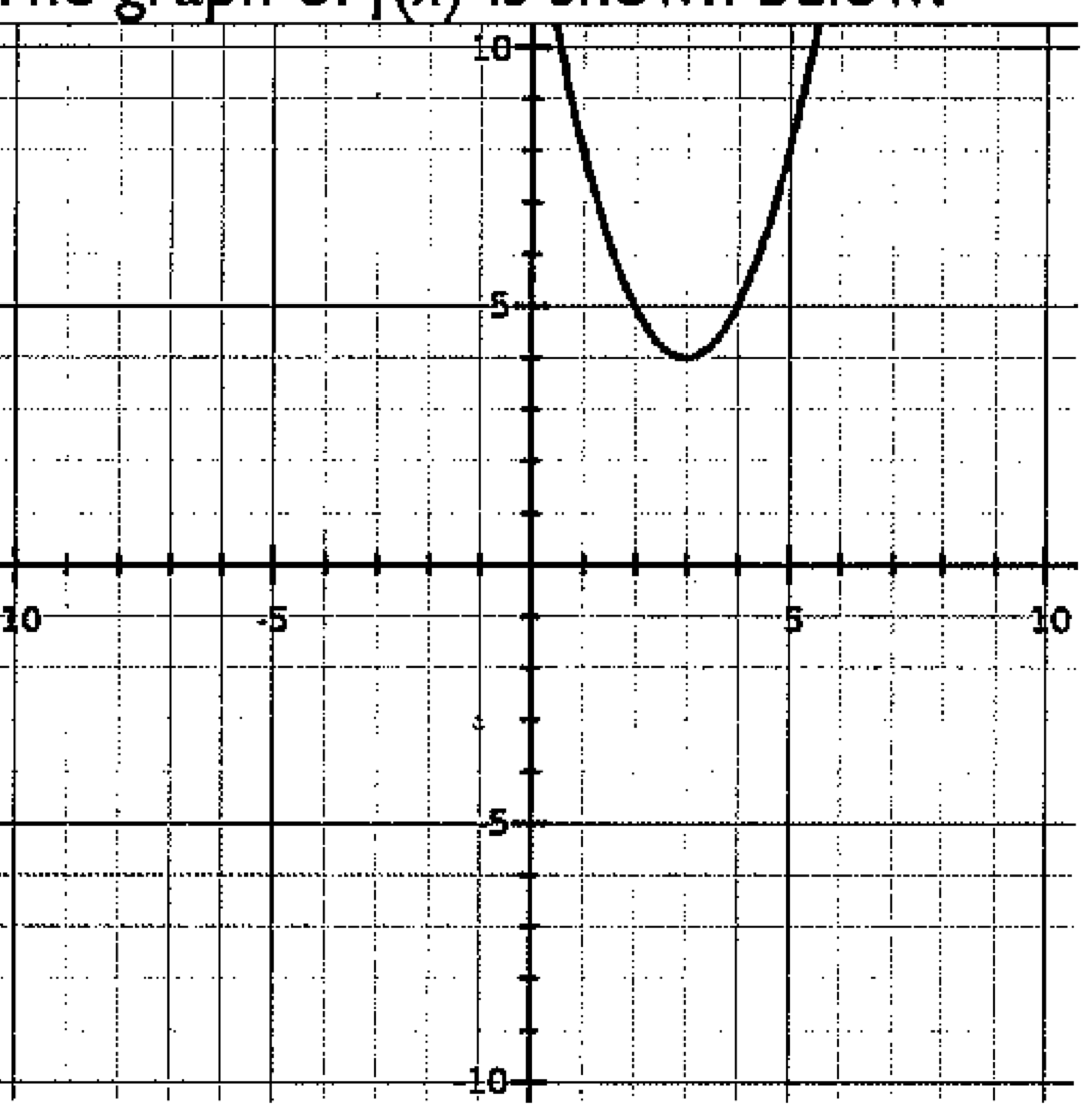
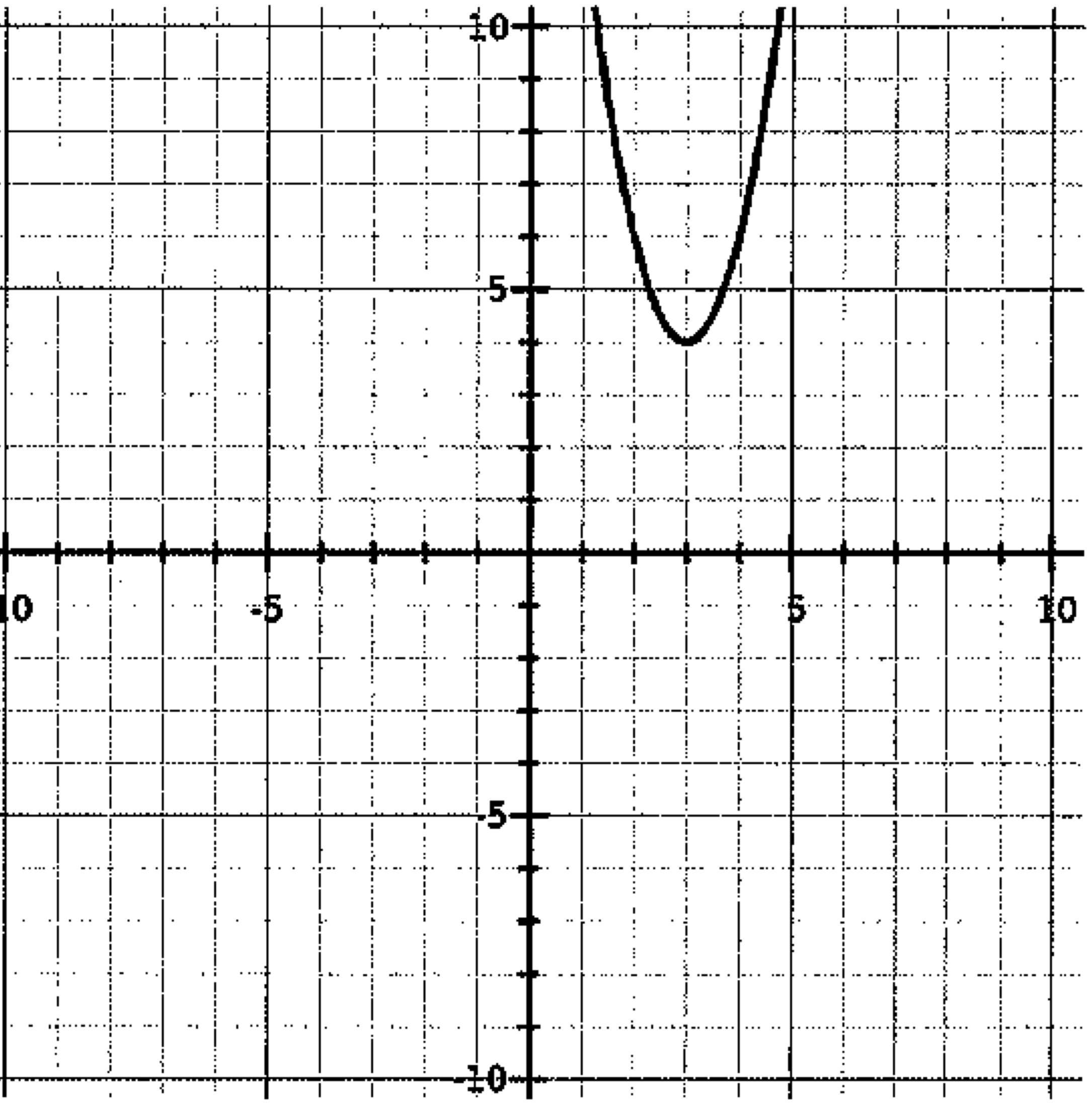
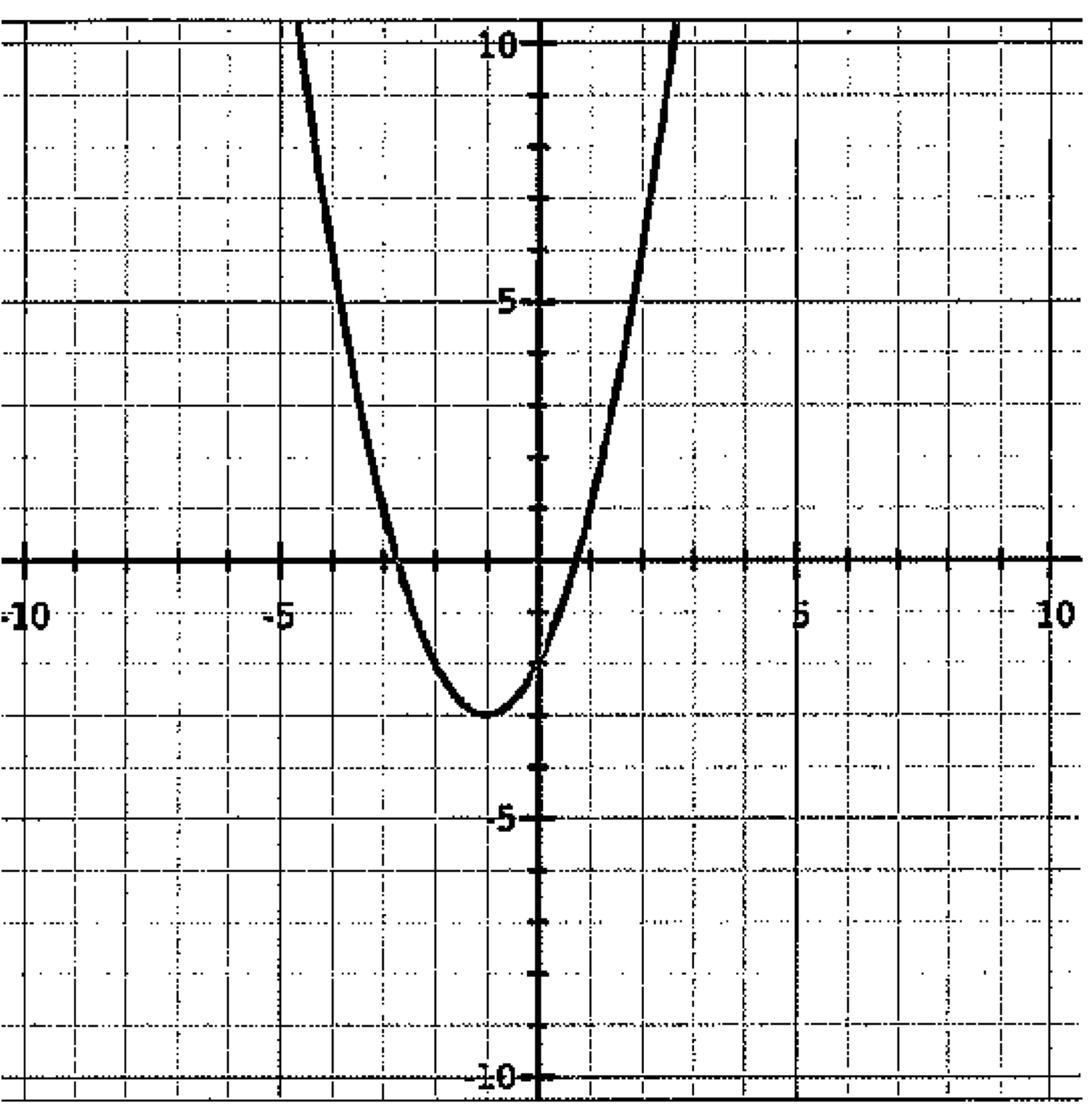
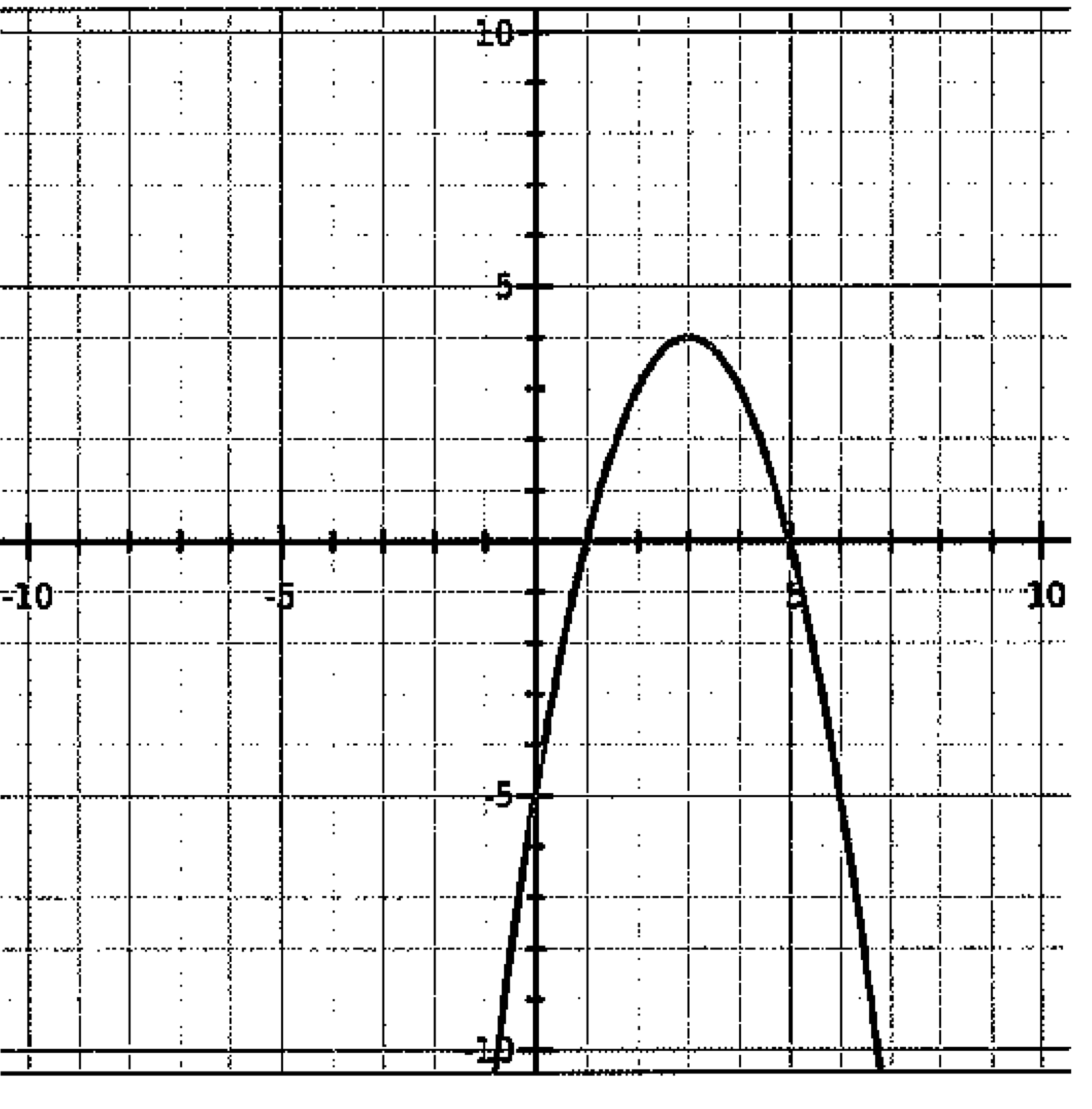
a)  b) 

1. Condense the following expressions:

a)  b) 

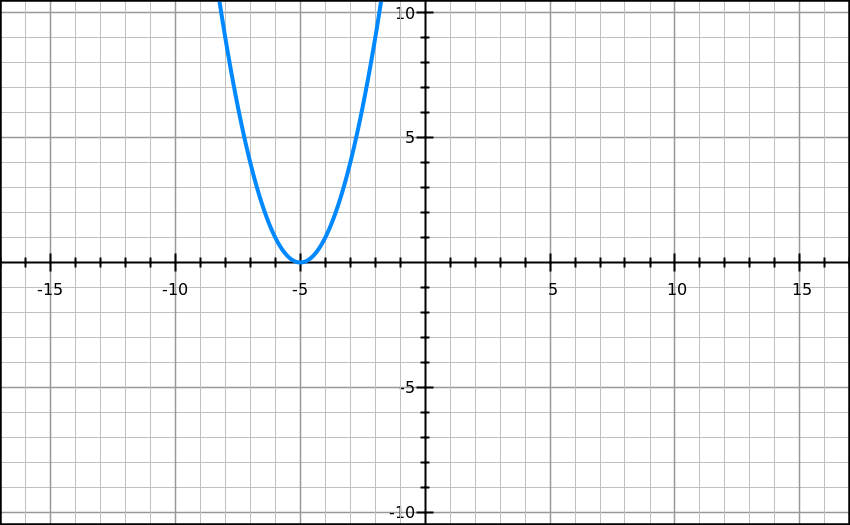
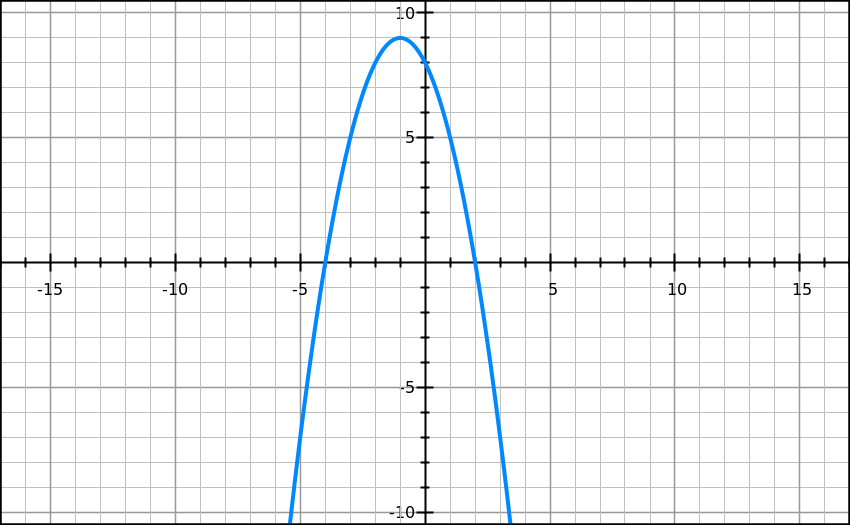
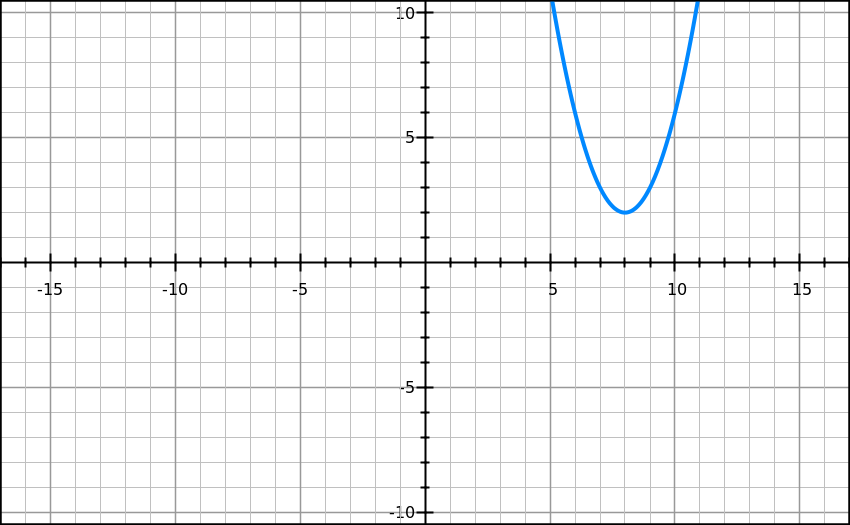
1. Solve for x. Round to the nearest hundredth if necessary.
2.  d) 
3.  e) 
4.  f) 

**QUADRATIC FUNCTIONS:**

1. 2x2 – 6x + 5 = 0
2. 4x2 – 2x – 4 = 2
3. The expression x4 – 64 can be rewritten in the following form where a, b, and c are real numbers:  
   (x2 + a)(x + b)(x + c). Find the values of a, b, and c.
4. x2 – 8x + 21 = (x – 4)2 + 3x – 16
5. The expression x2(x – y)3 – y2(x – y)3 can be written in the form (x – y)*a*(x + y), where *a* is a constant. What is the value of *a*?
6. Consider the function *f(x)* = (2x – 1)(x + 4)(x – 2). For what values of x is f(x) > 0?
7. Consider a quadratic equation with integer coefficients and two distinct zeroes. If one zero is irrational, which statement is true about the other zero?
   1. The other zero must be rational.
   2. The other zero must be irrational.
   3. The other zero can be either rational or irrational.
   4. The other zero must be non-real.
8. A softball was thrown from 1st to 2nd base. The height, in feet, of the ball above the ground *t* seconds after being thrown can be determined by the expression -16*t*2 + 40*t* + 4. What is the meaning of the 4 in the expression?
   1. The ball took 4 seconds to reach the ground.
   2. The ball took 4 seconds to reach its maximum height.
   3. The ball was thrown from a height of 4 feet.
   4. The ball reached a maximum height of 4 feet.
9. The graph of *f(x)* is shown below.  
     
   Identify the equations in the form *y* = *k(x – r) + n* which generate each of the graphs *a – c* as transformations of *f(x)*.   
   1. 
   2. 
   3. 
10. If *a* = 2 and *b* = 5, determine what condition(s) on *c* will restrict the solutions for *x* to real numbers.
11. Graph and find each of these characteristics for each parabola.   
     direction root(s) y-intercept vertex  
    1. 
    2. 
    3. 
    4. −2x+2  
       

1. You jump off a 9-meter diving board. At the same time, your friend jumps off a 10-meter diving board next to you. However, you and your friend have different styles of jumping. You jump up off the board with an initial speed of 1.6m/sec. Your friend just steps off the board, without jumping at all. The functions describing your heights off the surface of the water in terms of time (in seconds) are  
     
   You: Friend:
2. Who hits the water first?
3. When you jump in the air, do you ever get as high as the 10-meter board? If not, how high do you get?
4. Write the equation of a parabola in vertex form, standard form, and factored form (if possible) that has a maximum value of 8 *AND* a y-intercept at (0, 6).
5. Graph the parabola of . (2pts)  
   
6. Write the equation from problem #4 in vertex form. (1pt)
7. Graph the parabola of . (2pts).  
   
8. Write the equation from problem #6 in standard form (1pt.)

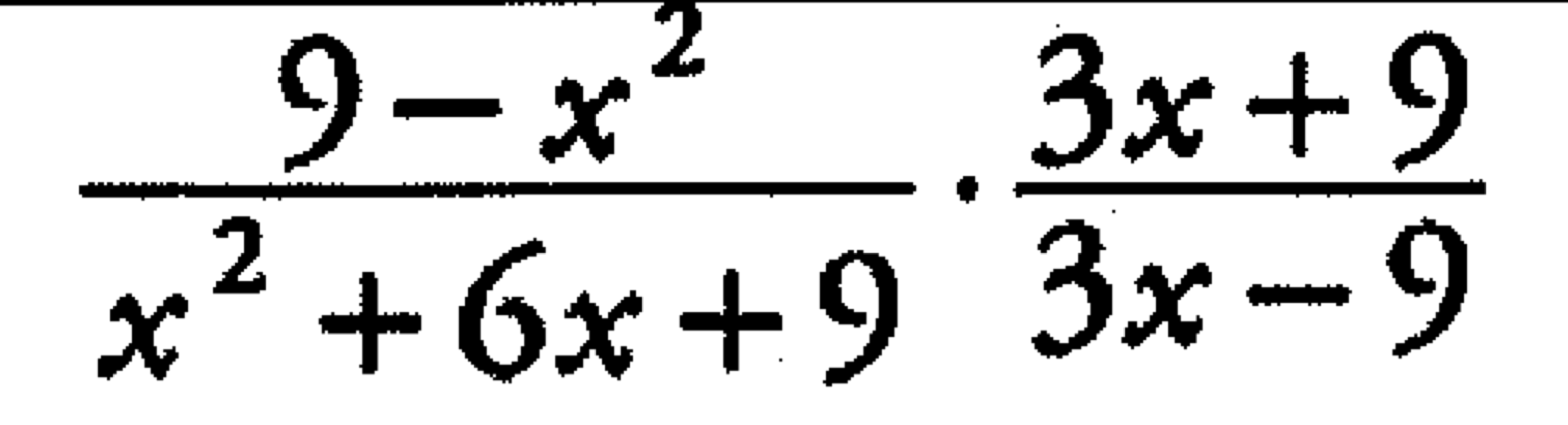
**Solve each equation. Leave all square roots in simplest radical form.**

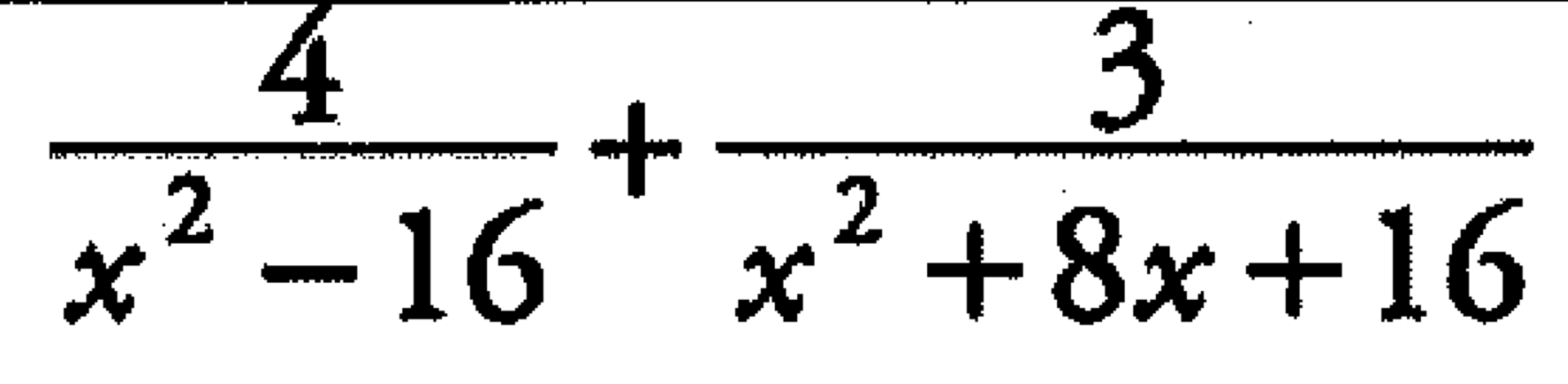
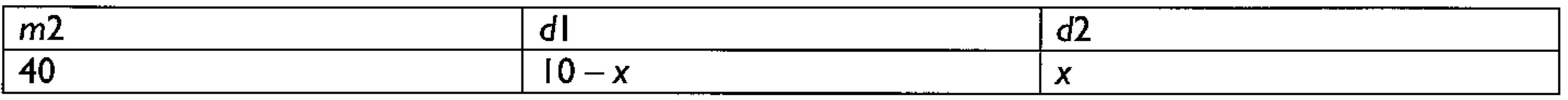
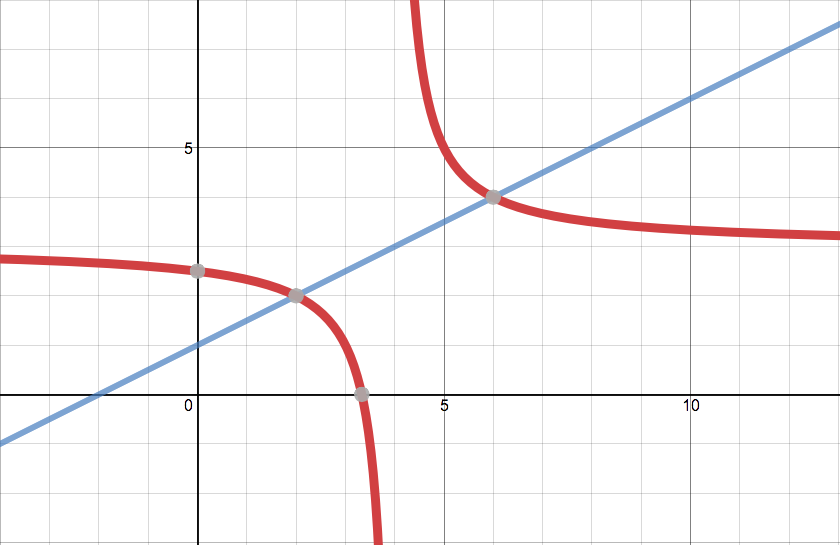
1. 7x²= 252
2. 2x² + 8x – 20 = 5x
3. 2*x²* + 4x + 3 = 0
4. Choose one graph below. Describe its discriminant as positive, negative, or zero, and tell how you know this.   
     
5. Which expression shows a correct use of the quadratic formula to solve the equation ?
7. What are the complex solutions to the following equation?

1. A polynomial is factored completely over the complex numbers.   
   Two of the factors are (*x* – 2*i*) and (*x* + 2*i*). Which expression could be the one that was factored?

**RATIONAL FUNCTIONS:**

1. Graph f(x) =
2. Simplify. Make sure to state any restrictions on the variables.



1. Simplify. Make sure to state any restrictions on the variable.  
   
2. For a seesaw to be balanced, *m1d1 = m2d2*. Use the information in the table to write the function *f(x)* that allows you to determine *m*1, the mass of the first child.  
   
3. What points are on the graph of the equation .
   1. (0, 16)
   2. (4, 0)
   3. (0, -2)
   4. (-2, 0)
4. A SmarTrip card on the Metro costs $5 for the card and then $2.10 for every Metro trip (assume for this problem that all trips cost the same amount). Write a function to show the average cost c of the SmarTrip card for *t* trips.
5. Identify the asymptotes of your function in the previous problem.
6. Write the four approach statements for your function in problem #1.
7. Graph y = .  
   
8. Rewrite y = to have asymptotes at x = -7 and y = 6.
9. Identify the vertical asymptotes, horizontal asymptotes, and holes of the graph of 
   1. Vertical asymptotes:
   2. Horizontal asymptotes:
   3. Holes:
10. Simplify and graph. Make sure to show all asymptotes and holes.   
    
11. Simplify and graph. Make sure to show all asymptotes and holes.   
    
12. Divide. Simplify your answer.   
    
13. Find the LCM of 2x + 4 and x² + 2x
14. Simplify
15. Simplify 
16. Simplify .
17. Solve 
18. Solve 
19. Write the equation of each of the functions shown below.  
    
20. Find the points of intersection of the two functions shown above. You must show algebraically how you arrived at your solution in order to earn credit.

**FUNCTION TRANSLATIONS:**

1. The function *f* is defined as *f(x)* = ¼(2)*x*. Write an expression that defines *f*(*x* + 3).
2. Given these functions:

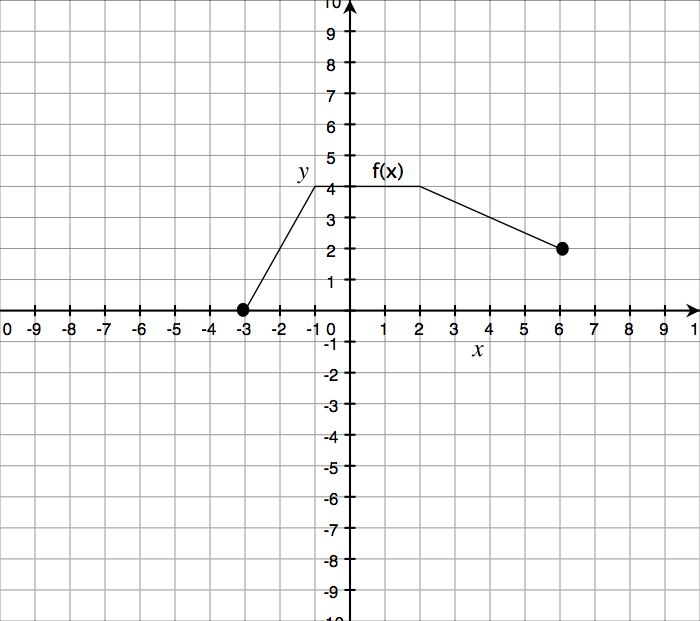
ADD(a, b) = a + b

SUBTRACT(a, b) = a – b

1. Find ADD(7,10)

1. Find SUBTRACT(2,-4)
2. Is ADD(2, ADD(x, -3)) = ADD(2x,-1)?
3. What is SUBTRACT(b, ADD(b, -b))?

3. Given the graph of f(x), graph g(x) and h(x).



g(x) = f(x + 3) + 5

h(x) = -2f(x)

4. Complete the table.

|  |  |  |  |
| --- | --- | --- | --- |
| **Location of Point on the Original Function**  **y = f(x)** | **Translated / Transformed Function** | **Type of translation / transformation**  **(Check all that apply)** | **Location of Point on the Translated Function** |
| (10, 10) |  | [ ] Vertical shift up by \_\_\_\_\_\_\_  [ ] Vertical shift down by \_\_\_\_\_\_\_  [ ] Horizontal shift left by \_\_\_\_\_\_\_  [ ] Horizontal shift right by \_\_\_\_\_  [ ] Vertical stretch by \_\_\_\_\_\_  [ ] Vertical compression by \_\_\_\_\_  [ ] Reflection over the x-axis |  |
| (-5, 6) |  | [ ] Vertical shift up by \_\_\_\_\_\_\_  [ ] Vertical shift down by \_\_\_\_\_\_\_  [ ] Horizontal shift left by \_\_\_\_\_\_\_  [ ] Horizontal shift right by \_\_\_\_\_  [ ] Vertical stretch by \_\_\_\_\_\_  [ ] Vertical compression by \_\_\_\_\_  [ ] Reflection over the x-axis |  |

5. Describe the translation of each graph from f(x) to g(x).

f(x) = x2 g(x) = (x – 12)2 + 6

f(x) = |x – 5| + 1 g(x) = |x + 2| – 8

6.Evaluate and simplify each expression.

Let , , , 

**  **

**** ** **

7. Evaluate and simplify each expression.

**f:**

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **x** | 5 | -5 | 3 | 8 | 0 | 1 | 9 |
| **y** | 12 | 10 | -1 | 3 | 0 | -5 | 7 |

**g:**

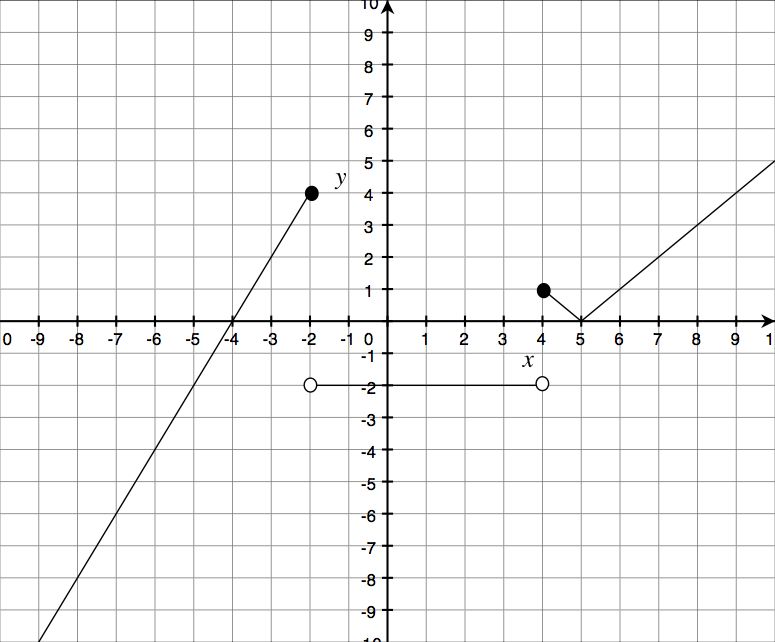
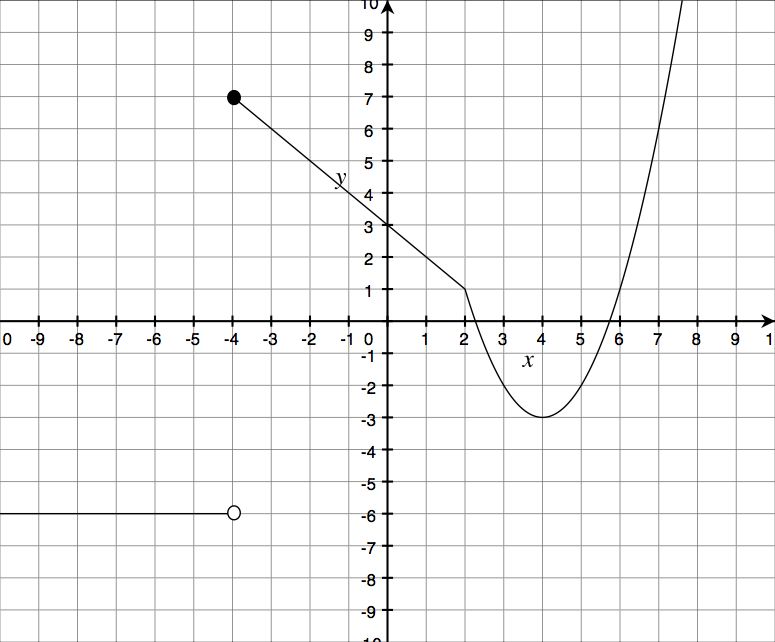
|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **x** | -3 | 2 | 8 | 1 | -4 | 7 | 6 |
| **y** | 0 | -5 | -5 | 8 | 1 | 3 | 5 |

f(5) = g(1) = f(-5) – g(2) =

f(1)g(1) = [f(9) + g(8)]f(8) = f(g(8)) = g(f(9)) =

8. Evaluate each expression.

f(-4) = f(4) = g(-2) = f(-20) =



*f(x)*

*g(x)*

f(g(1.5)) = g(f(4)) = f(g(4)) = (f + g)(7) =

9.

Let , , .

Find each of the following, and simplify.

1) 

2) 

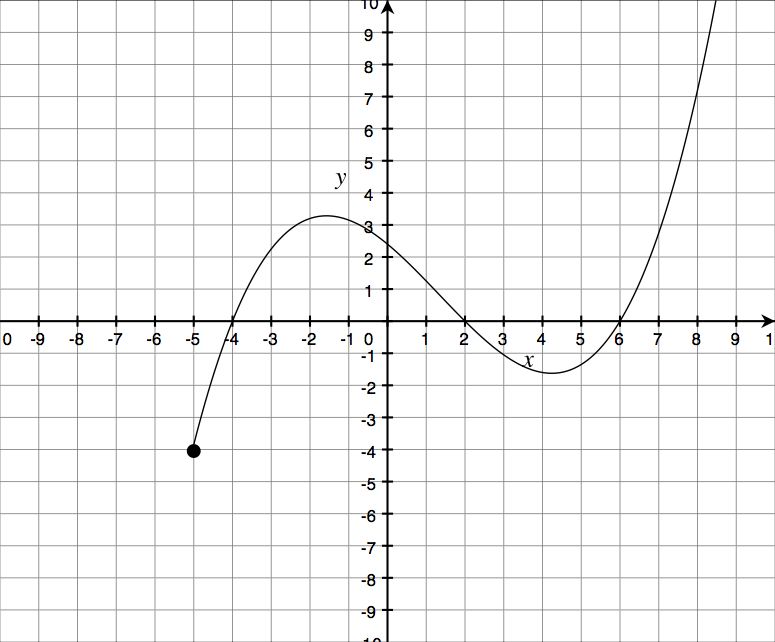
3) 

4) 

5) =

**Part 3: Graphical Analysis**

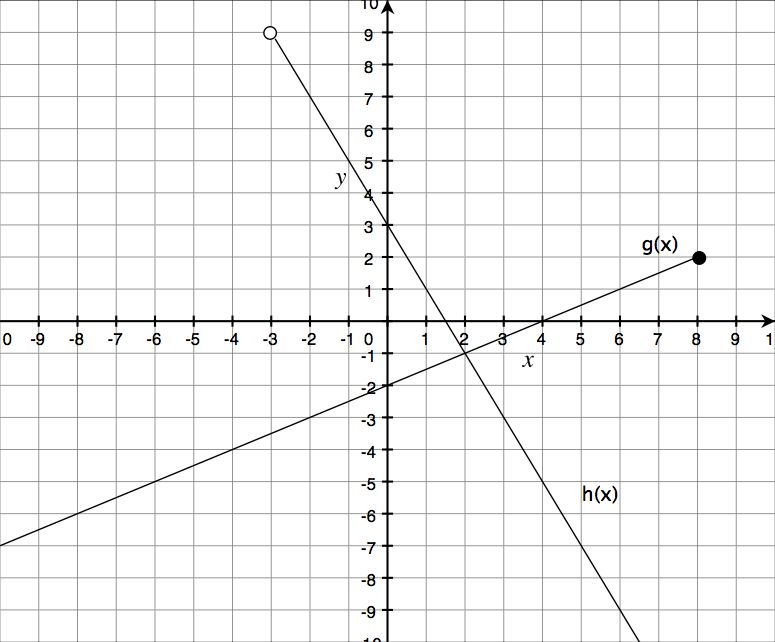
*f(x)*



*f(x)*

1. What is the domain of f(x)?
2. What is the range of f(x)?
3. For what values of *x* does f(x) = 0?
4. What is the value of f(x) when *x* = 0?
5. Over what intervals is f(x) > 0?

1. What is the domain of g(x)?

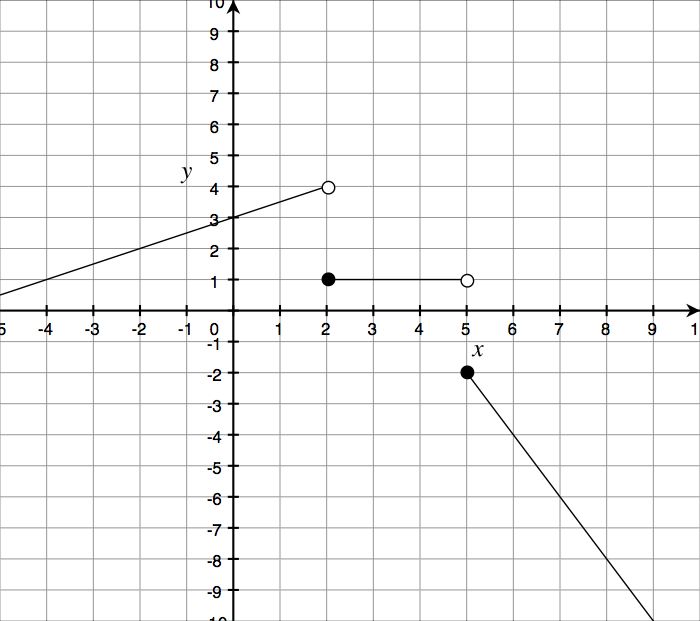


1. What is the range of g(x)?
2. What is the domain of h(x)?
3. What is the range of h(x)?

10)  11)  12)  13) 

**Part 4: Piecewise Functions**

Write the piecewise function f(x) that would produce the following graph:



Graph f(x) on the coordinate plane. Then, find **exact** answers for the evaluation problems that follow.

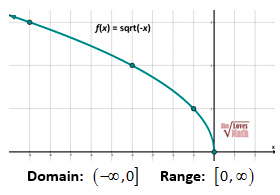






9. Determine whether the two functions are inverses. State how you know.   


10. Use the following graph to answer the questions.  
 

  
a) Draw *f(x)* rotated 180° about the origin, and state the equation of the transformed function.

b) Draw *f(x)* reflected across the y-axis, and state the equation of the transformed function.



c) Draw *f(x)* reflected across the x-axis, and state the equation of the transformed function





**SYSTEMS OF EQUATIONS WITH FUNCTIONS:**

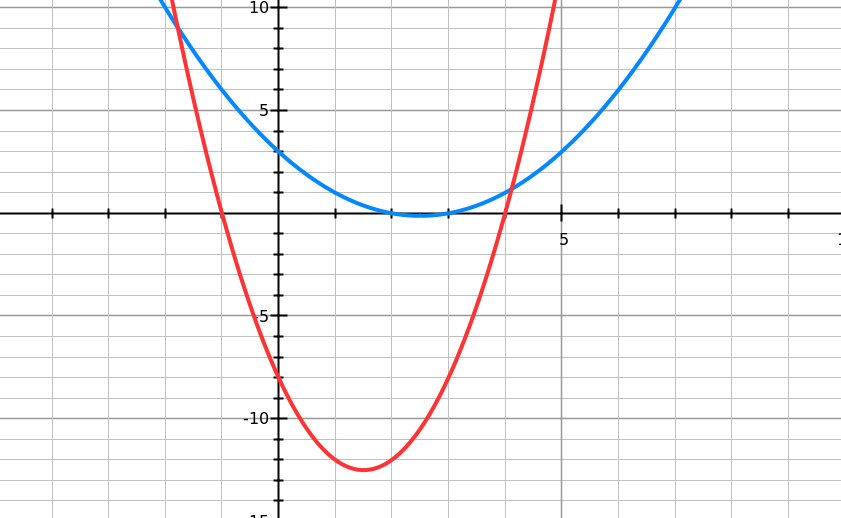
1. Functions *f* and *g* are defined below.  
   The graphs of *y = f(x)* and *y = g(x)* intersect at point *P*. Determine the *x*-coordinate of *P*. Leave your answer in simplest radical form.
2. Find the solution to this system of equations:

-5*x* + *y* = 12  
-3(*x* – 2*y*) = 4



**Directions:**

1. Write the equation of each parabola in **each** form (**standard**, **vertex**, *and* **factored**).
2. Identify the **vertex** and **zeroes/x-intercepts** of each function.
3. Find the points of intersection. Leave all irrational numbers in simplest radical form.



**Directions:**

1. Draw the graphs of each of the following three functions on the graph below.
2. Identify the **vertex** and **zeroes/x-intercepts** of each function.
3. Find the points of intersection. Leave all irrational numbers in simplest radical form.

